



# Coupling of temperatures and power outputs in hybrid photovoltaic and thermoelectric modules



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## ABSTRACT

The hybrid system of a photovoltaic solar-cell module and a thermoelectric module has drawn considerable attention recently. Here, unlike most of the previously reported work, we present an analysis that intimately couples the nodal temperatures of the system and power outputs of both modules. Within the range of selected solar constants, we have obtained efficiencies for hybrid systems in comparison with those of photovoltaic solar cells alone. Our findings indicate that, for parametric values chosen herein, the former are generally lower than the latter, unless thermal electric materials of low-valued thermal conductivities and high-valued Seebeck coefficients can be found or fabricated.

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## 1. Introduction

Photovoltaic solar cells are devices that convert solar energy into electricity.

During the conversion process, a portion of the impinging radiation is absorbed by the cells, causing the rise of the photovoltaic temperature. It is known [1] that, for photovoltaic cells, the output power decreases as the temperature increases. Therefore, if we are able to design a hybrid system consisting of a solar-cell unit and a thermoelectric module, allowing the latter to dissipate the absorbed energy in the former, and at the same time to reduce the photovoltaic temperature, there may be some hope to produce a larger amount of the output electric power than the solar-cell module alone.

During recent decades, advantages of thermoelectric devices have gradually been realized because these devices are capable of directly converting the thermal energy into the electricity within small sizes. Characteristics of thermoelectric modules alone were investigated [2–4]. In [5], researchers started focusing on heat dissipation of a hybrid system. A couple of years later, van Sark [6] adopted a new type of thermoelectric material when designing

the device. A hybrid system with water cooling was also studied [7]. Using heat flux concentrators, Vorobiev et al. [8] developed a hybrid system that yielded high efficiencies. In [9], evacuated-tube heat-pipe solar collectors were studied, with agreeable comparisons of theoretical results and experimental data. A computational study [10] using the finite element method was conducted for a hybrid system with a collector panel inserted between the photovoltaic cell and the thermoelectric module, yielding high thermal efficiencies. Finally, in [11], the thermoelectric module was computationally investigated, with the solar flux input as one of the boundary conditions.

Here in this analysis we have placed the emphasis on the coupling between discretized nodal temperatures in the heat conduction phenomenon and the electrical power outputs produced by the hybrid system. In other words, nodal temperatures are functions of power outputs, and vice versa. Therefore,  $T(i)$ ,  $P_{te}$ , and  $P_{pv}$  are strongly inter-related. The thermoelectric efficiency,  $\eta_{te}$ , is not assumed to be the maximum value. These variables must be computed simultaneously. Such a coupling approach constitutes a nonlinear challenge, and is presented below.

## 2. Modeling and formulations

The system schematic of an assembly consisting of a photovoltaic solar unit and a slab of thermoelectric material is shown in Fig. 1. The sunlight impinges upon the top surface of the assembly.

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### Nomenclature

$A_c$	area of the hybrid system ( $\text{m}^2$ )	$P_j$	energy of Joule heating (W)
$bi_1$	Biot number defined as $h_1\Delta y_{te}/k_{te}$ for bottom face	$P_{pv}$	power output of photovoltaic solar cells (W)
$bi_5$	Biot number defined as $h_5\Delta y_g/k_g$ for top face	$P_{te}$	power output of thermoelectric module (W)
$c$	speed of light, $3 \times 10^8(\text{m/s})$	$P_{hybrid}$	power output of the hybrid system (W)
$d_1, d_2$	empirical constants given in Eq. (6)	$Rr$	radiative recombination rate at zero quasi-Fermi level (same unit as $N_{gap}$ )
$e$	elementary charge $1.6 \times 10^{-19}(\text{C})$	$R, r$	external and internal resistances of thermoelectric module circuit ( $\Omega$ )
$E$	energy of a solar photon (eV)	$S$	Seebeck coefficient of thermoelectric module ( $\mu\text{V/K}$ )
$E_g$	bandgap of silicon material (eV)	$Sc$	solar irradiance (solar constant) ( $\text{W/m}^2$ )
$h$	Planck constant $6.63 \times 10^{-34}(\text{J s})$	$T_1, \dots, T_5$	nodal temperatures at node 1, ..., 5 (K)
$h_1$	heat transfer coefficient for the bottom surface ( $\text{W/m}^2 \text{K}$ )	$T_{air}$	temperature of the ambient air (K)
$h_5$	heat transfer coefficient for the top surface ( $\text{W/m}^2 \text{K}$ )	$T_{surr}$	temperature of the surroundings (K)
$I$	electrical current in thermoelectric module (A)	$V_{te}$	voltage of the thermoelectric module circuit (V)
$I_{pv}$	electrical current in the solar cell (A)	$V_{pv}$	external voltage of the solar cell (V)
$k_{te}$	thermal conductivity of the thermoelectric module ( $\text{W/m K}$ )	$\Delta y_{te}$	half thickness of the thermoelectric module (m)
$k_{sub}$	thermal conductivity of the substrate in solar cells ( $\text{W/m K}$ )	$\Delta y_{sub}$	half thickness of the substrate (m)
$k_g$	thermal conductivity of the glass layer in solar cells ( $\text{W/m K}$ )	$\Delta y_g$	half thickness of the glass layer (m)
$n$	number of p–n pairs in the thermoelectric module	<b>Greek symbols</b>	
$\dot{N}_E$	$\dot{N}_{gap}$ per unit electron volt	$\alpha, \beta$	constants related to bandgap and given in Eq. (8)
$\dot{N}_{gap}$	number of solar photons per unit time per unit area, carrying energy larger than the bandgap	$\varepsilon$	emissivity of the protective glass layer
$q_s$	heat flow rate entering the control volume (W)	$\rho$	reflectivity of the protective glass layer
$q_n$	heat flow rate leaving the control volume (W)	$\sigma$	Stefan–Boltzmann constant ( $5.67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)$ )
		$\eta_{hybrid}$	efficiency of the hybrid system

A sheet of photovoltaic thin film is imbedded inside the unit filled with the protective glass and the substrate material, respectively, above and below the film. Underneath the solar-cell unit, a thermoelectric module is attached. Both of the bottom surface and the top surface of the assembly are cooled by air flows.

The system is assumed to be in steady state, and the heat transfer phenomenon takes place in the one-dimensional manner, with all variables being functions of the  $y$  coordinate only. Based on the first law of thermodynamics, we can derive five nonlinear algebraic equations governing  $T_1, T_2, T_3, T_4$ , and  $T_5$ , as shown in Fig. 1. Even though we are aware that heat fluxes flow downward in real-life situations, we will proceed to assume that they flow upward along the positive  $y$  direction to avoid sign confusions.

For  $T_1$ ,

$$A_c h_1 (T_{air} - T_1) = A_c k_{te} (T_1 - T_2) / \Delta y_{te} + P_{te}(1), \quad (1)$$

which can be simplified and rearranged, with final coefficients listed in Table 1. The power output,  $P_{te}(1)$ , can be expressed by

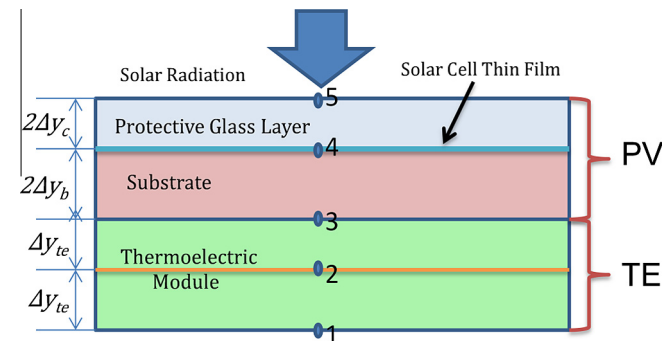


Fig. 1. The system schematic representing the hybrid system of photovoltaic solar cell and thermoelectric module.

$$P_{te}(1) = V(1)I - P_j(1), \quad (2a)$$

in which the Seebeck electrical current,  $I$ , is needed. The relationship between the current and the Seebeck voltage can be expressed by

$$I = (V(1) + V(2) + V(3)) / (R + r(1) + r(2) + r(3)), \quad (2b)$$

$$P_j(1) = I^2 r(1), \quad (2c)$$

$$\text{where } V(1) = nS(T_{1.5} - T_1), \quad T_{1.5} = (T_1 + T_2)/2, \quad (2d)$$

$$\text{and } r(1) = e_1(r_0 + \beta T_1). \quad (2e)$$

Values of  $e_1, r_0$ , and  $\beta$  are given in Appendix A. Definitions of voltages, internal electrical resistance, and midpoint nodal temperatures at other nodes can also be found in Appendix A. The constant  $n$  is the number of p–n semi-conductor pairs, and  $S$  is the Seebeck coefficient. It can be observed that, via Eqs. (1, 2a–e), the electrical current and the nodal temperatures are strongly coupled.

For the derivation of the governing equation for  $T_2$ , the control volume, over which the energy conservation is taken, is shown in the dashed line in Fig. 2. The nodal temperature  $T_2$  is situated at the center of the control volume. From south, the heat flux enters the control volume. It leaves the control volume to north. The Joule heating can be treated as a heat sink inside the control volume, and does not explicitly appear in the energy-conservation equation. In addition, there is power output due to Seebeck thermoelectric effects. These quantities are designated, respectively, as  $q_s, q_n$ , and  $P_{te}$ . Consequently, according to the flux directions indicated by arrows, we obtain

$$q_s - q_n - P_{te}(2) = 0, \quad (3)$$

where

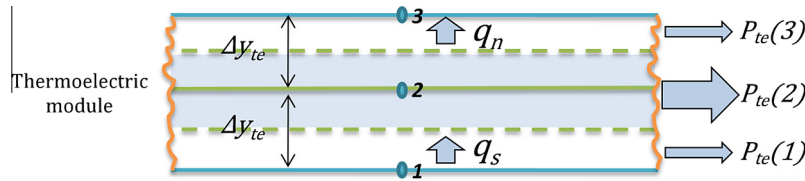
$$q_s = A_c k_{te} (T_1 - T_2) / \Delta y_{te}, \text{ and } q_n = A_c k_{te} (T_2 - T_3) / \Delta y_{te}.$$

**Table 1**

The matrix coefficients for the nonlinear set of 5 equations governing nodal temperatures, power output of the thermoelectric module, and power output of the photovoltaic solar cell, along with the constant vector.

$$a = \begin{bmatrix} -1 - bi_1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & -k_{te}/\Delta y_{te} & -k_{te}/\Delta y_{te} - k_{sub}/\Delta y_{sub} & k_{sub}/\Delta y_{sub} & 0 \\ 0 & 0 & k_{sub}/\Delta y_{sub} & -k_{sub}/\Delta y_{sub} - k_g/\Delta y_g & k_g/\Delta y_g \\ 0 & 0 & 0 & 1 & -1 - bi_5 \end{bmatrix}$$

$$b = \begin{bmatrix} P_{te}(1)\Delta y_{te}/(k_{te}A_c) - bi_1 T_{air} \\ P_{te}(2)\Delta y_{te}/(k_{te}A_c) \\ P_{te}(3)/A_c \\ P_{pv}/A_c \\ [-S_c + \epsilon \sigma (T_5^4 - T_{surr}^4) + \rho S_c] \Delta y_g/k_g - bi_5 T_{air} \end{bmatrix}$$



**Fig. 2.** A typical control volume containing nodal temperature,  $T_2$ . It is noted that  $P_j$  and  $P_{te}$  should bear the same sign.

The thermoelectric power output,  $P_{te}(2)$ , can be obtained in a similar manner as Eq. (2a).

In the derivation of the governing equation for  $T_3$ , which is located at the interface between the thermoelectric material and the photovoltaic module, we idealize the thermal contact resistance to be zero. After straightforward algebraic manipulations, we obtain

$$a(3,2)T_2 + a(3,3)T_3 + a(3,4)T_4 = b(3), \quad (4)$$

where the coefficients are listed in Table 1.

The derivation of the governing equation for  $T_4$  is similar to that for  $T_3$ . The final equation can also be cast into the standard form as

$$a(4,3)T_3 + a(4,4)T_4 + a(4,5)T_5 = b(4). \quad (5)$$

It is worth noting that the electrical power delivered by the photovoltaic film can be related to the nodal temperature by

$$P_{pv} = A_c(d_1 + d_2 T_4), \quad (6)$$

where  $d_1$  and  $d_2$  are constants, whose values depend on the impinging solar constants, and can be computed *a priori* independently of the system characteristics. Such a linear approximation greatly simplifies the analysis, enabling us to officially couple the nodal temperatures and the power outputs.

The details are briefly given below.

First, the number of solar photons per unit time per unit area, carrying energy (that is larger than the bandgap) per electron volt can be given as

$$\dot{N}_E = (AM1.5)(hc/E^3), \quad (7)$$

where AM1.5 stands for NREL's (National Renewable Energy Laboratory) data for the solar spectrum at  $S_c = 1000 \text{ W/m}^2$ , with its unit being  $\text{W/m}^2 \text{ nm}$ ;  $h$  represents the Planck constant,  $6.63 \times 10^{-34} \text{ J s}$ ; the symbol,  $c$ , is the speed of light,  $3 \times 10^8 \text{ m/s}$ .

Next, according to [12], the bandgap,  $E_g$ , decreases with increasing temperatures, and can be written as

$$E_g(T) = E_g(0) - \alpha T^2 / (T + \beta), \quad (8)$$

where  $E_g(0) = 1.1557 \text{ eV}$ ,  $\alpha = 7.021 \times 10^{-4} \text{ eV/K}$ , and  $\beta = 1108 \text{ K}$ .

Only photons that carry the energies greater than the bandgap are capable of exciting the electrons located in the valence band, and force them to jump to the conduction band. The number of such photons can be expressed by

$$\dot{N}_{gap} = \int_{E_g}^{\infty} \dot{N}_E dE = \int_{E_g}^{\infty} (AM1.5) \left( \frac{hc}{E^3} \right) dE = \text{function of } T. \quad (9)$$

A portion of the excited electrons may return to the valence band, and recombine with holes. For such recombination activities, the rate can be derived as

$$Rr = \frac{2\pi}{c^2 h^3} \int_{E_g}^{\infty} \frac{E^2 dE}{\exp[E/(k_B T)] - 1}, \quad (10)$$

thus inviting Eq. (8) to enter the problem [13]. The electrical current is generated from the electron-hole pairs that do not recombine. Note that the only recombination mechanism considered in this analysis is caused by radiation. Under this idealization, we can write

$$I_{pv} = e\dot{N}_{gap} - eRr \{ \exp[eV_{pv}/(k_B T)] - 1 \}, \quad (11)$$

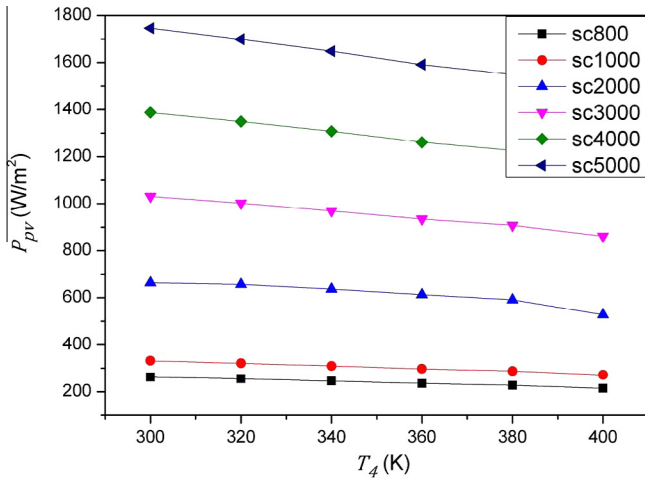
where the internal series resistance and the shunt resistance of solar cells have been neglected. Finally, the output power of the solar cell can be expressed by

$$P_{pv} = V_{pv} I_{pv} = \text{function of } V_{pv}. \quad (12)$$

Under the assumption that solar cells deliver the maximum power output, we can numerically find the corresponding maximum power output from Eq. (12). The laborious procedure of finding  $d_1$  and  $d_2$  can be briefly described as (a) give a value of  $T$ ; (b) compute

**Table 2**  
Constant values of  $d_1$  and  $d_2$  for various values of  $Sc$ .

$Sc(W/m^2)$	$d_1$	$d_2$
800	440.070	−0.579
1000	510.337	−0.592
2000	1064.852	−1.287
3000	1532.460	−1.661
4000	2043.820	−2.173
5000	2555.176	−2.677
6000	3066.575	−3.173



**Fig. 3.** Power output of photovoltaic solar cell versus the temperature of the thin-film solar cell, parametrized in solar constants.

$E_g$  according to Eq. (8); (c) find maximum  $P_{pv}$ ; finally (d) use the curve fitting to obtain  $d_1$  and  $d_2$ . Table 2 lists their values for  $Sc = 800\text{--}6000\text{ W/m}^2$ .

In Fig. 3, a family of nearly linear lines of  $P_{pv}$  versus the photovoltaic temperature are plotted, parameterized in six various values of  $Sc$  for silicon solar cells. Finally, it is also noted that we assume the film to be so thin like a sheet (of a few nanometers) that its thickness is negligible in comparison with those of thermoelectric module, the substrate, and the protective glass layer.

For the derivation of the governing equation for  $T_5$ , we take the energy balance over the half-grid-interval control volume between  $x_{4.5}$  (the average of  $x_4$  and  $x_5$  and the top surface. This balance dictates that the northbound heat conduction due to the temperature gradient between node 4 and node 5 should be equal to the energy leaving the sheet (thus, energy components entering the sheet should be considered negative), eventually yielding

$$T_4 - (1 + bi_5)T_5 = c_5 \Delta y_{sub} / k_{sub}, \quad (13)$$

where  $c_5$  includes energy components of radiation and convection entering and exiting above the top surface, and is defined as

$$c_5 = q_{r,net} + (\rho - 1)Sc - h_5 T_{air}. \quad (14)$$

In Eq. (14), the first term denotes the net radiation loss from the system to the surroundings, and is written as

$$q_{r,net} = \epsilon \sigma (T_5^4 - T_{surr}^4). \quad (15)$$

Equation (15) holds under the assumption that the area of the absorbing top surface of the hybrid system is much smaller than that of the surroundings, such as the sky. In the present analysis, this assumption is certainly valid. In addition, it should be made clear that the temperature of the surroundings, such as clouds in the sky, may not necessarily be the same as that of the ambient air near the earth ground.

Equations (1), (2a)–(2e), (3)–(15) constitute a set of nonlinear equations governing the characteristics of the assembly system. When the solution converges, the efficiency of the hybrid system can be defined and computed by

$$\eta_{hybrid} = P_{hybrid} / Sc = [P_{te}(1) + P_{te}(2) + P_{te}(3) + P_{pv}] / Sc. \quad (16)$$

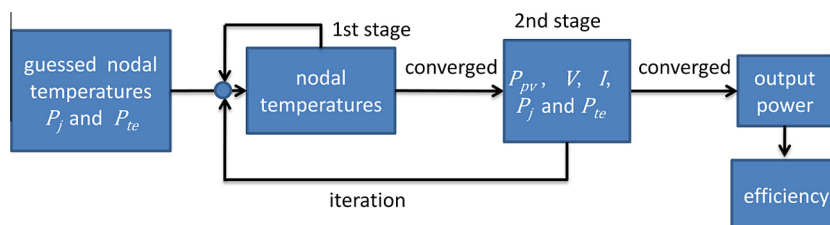
The solution procedure is described in the next section.

### 3. Computational procedure

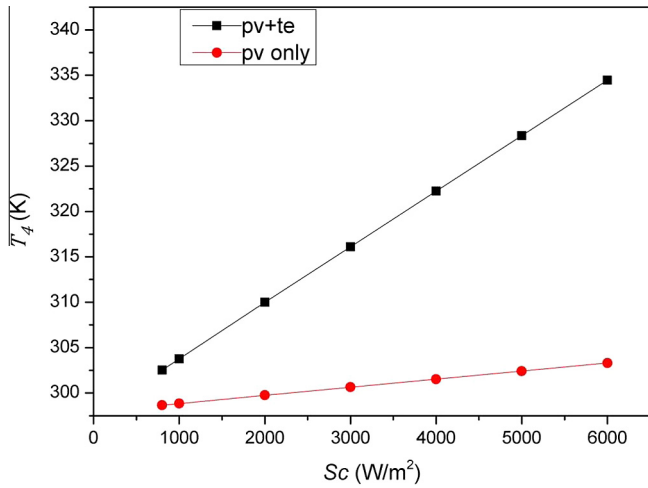
Equations (1), (2a)–(2e), (3)–(16) can be solved using the standard Newton–Raphson (NR) method [14]. However, we are concerned that, after interspersing linearized terms (related to photovoltaic and thermoelectric behaviors) with linear terms discretized from the heat conduction, we may encounter difficulties in debugging the code in the event of divergences. Therefore, we have opted the adoption of a modified version of NR method, in which two stages of computations are conducted. In the first stage, nonlinear terms are left in the right-hand side of equations. For the sake of convergence safety, the linearized solution obtained by using a direct linear solver is preferably under-relaxed. In the second stage, all nonlinear terms are computed using the newly iterated and under-relaxed values. The procedure then repeats.

It is understood that this two-staged under-relaxed NR version is clearly not as robust as the standard NR version, and its convergence rate is also slower than the standard NR counterpart. The advantage of this modified version, however, is twofold: (1) a great amount of labor associated with algebraic manipulations and coding is saved, and (2) debugging the code is easier when divergence happens. Regardless of choices, the two solutions computed using these two methods should become identical upon solution convergence. The concept of this mathematical modeling is similar to that of the compartment modeling widely used in ecological sciences [15].

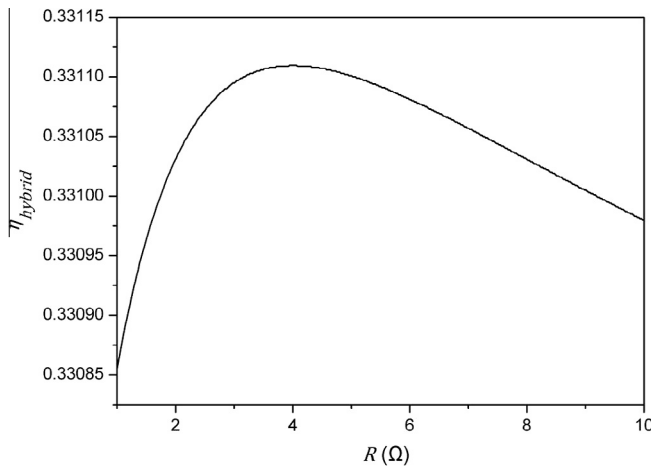
Fig. 4 shows the flow chart of the computer code. In the first stage, nodal temperatures are computed iteratively, with power outputs of the hybrid system either guessed or previously iterated. Iterations are needed because the radiation term,  $T^4$ , exists in Eqs. (13)–(15). In the second stage, these more accurate nodal temperatures are used to update values of internal electrical resistances in



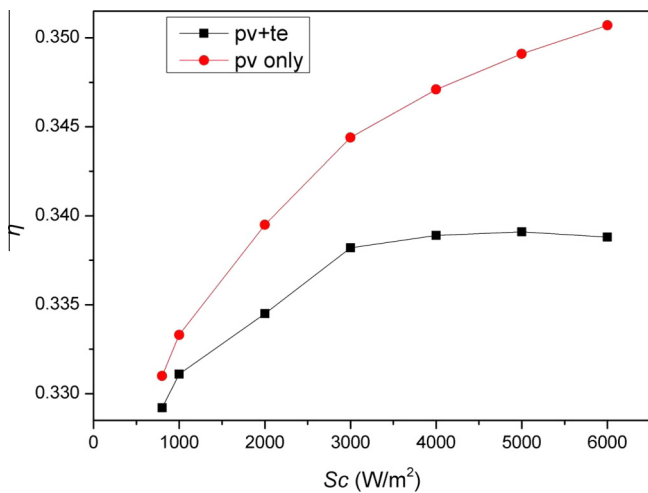
**Fig. 4.** Flowchart of the algorithm that is used to iteratively compute nodal temperatures, power output of the thermoelectric module, and power output of the photovoltaic solar cell.



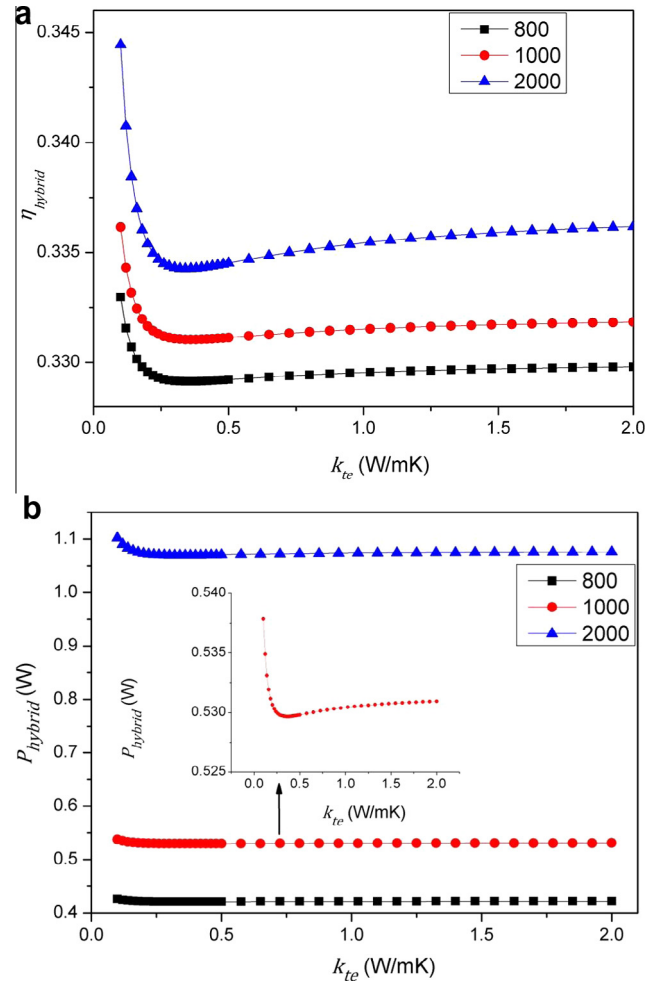
**Fig. 5.** Comparison of the temperature of the thin-film solar cell for two cases: (a) photovoltaic solar cell only, and (b) the hybrid system.



**Fig. 6.** Efficiency versus external electrical resistance. The  $R$  value that corresponds to the maximum efficiency is chosen for computations.



**Fig. 7.** Comparison of efficiencies for two cases: (a) photovoltaic solar cell only ( $\eta = P_{pv}/Sc$ ), and (b) the hybrid system.



**Fig. 8.** (a) Efficiency of the hybrid system versus the thermoelectric thermal conductivity, and (b) power output of the hybrid system versus the thermoelectric thermal conductivity. Both cases are parametrized in  $Sc$ .

the thermoelectric module and these power outputs. For all runs, the global energy balance has been checked to ensure the correctness of the code.

#### 4. Results and discussion

Fig. 5 shows the photovoltaic temperature,  $T_4$ , versus the solar constant,  $Sc$  for both cases of (a) the photovoltaic thin film only and (b) the hybrid assembly of photovoltaic thin film and thermoelectric module. Both temperatures are almost linear functions of  $Sc$ . They are, however, associated with different slopes. It is observed that values of the hybrid case are overall higher than those of the thin film case, suggesting that the addition of the thermoelectric module blocks the cooling effect. As  $Sc$  increases, the difference in two temperatures also increases.

To choose an appropriate value of external electrical resistance,  $R$ , we have first evaluated the efficiencies of the hybrid system corresponding to various values of given  $R$ . Then the value that yields the maximum efficiency is chosen, as shown in Fig. 6. For example, for a typical run with the following data:

$Sc = 1000$  W/m<sup>2</sup>,  $k_{te} = 0.48$  W/(m K),  $n = 100$ ,  $L_{te} = 0.003$  m,  $L_{sub} = 0.003$  m,  $L_{glass} = 0.003$  m,  $r = 3.9973 \Omega$  (computed), and  $R = 3.9925 \Omega$ ,

the efficiency of the hybrid system is equal to 0.3311.



In Fig. 7, we present the efficiency,  $\eta$ , versus the solar constant,  $Sc$  for both cases of (a) the photovoltaic thin film only and (b) the PV–TE hybrid assembly. Both curves exhibit the trend that the efficiency increases as the solar constant increases as expected. Within the range of the investigated  $Sc$  values, the efficiency of the hybrid system is lower than that of the photovoltaic solar cell alone. This finding offers a warning guide for the industrial manufacturers who may be interested in fabricating such hybrid systems, with the hope that the hybrid systems will yield higher efficiencies.

Finally, Fig. 8a and b depict the efficiency,  $\eta_{\text{hybrid}}$  and the combined power output,  $P_{\text{hybrid}}$  versus the thermal conductivity of thermoelectric module,  $k_{te}$ . We note that both  $\eta_{\text{hybrid}}$  and  $P_{\text{hybrid}}$  decrease as  $k$  increases for  $k_{te} \leq 0.25$ . Beyond this  $k_{te}$  value, both the efficiency and the power output remain fairly unchanged. Another noteworthy feature is that the sensitivity of  $\eta_{\text{hybrid}}$  on  $k_{te}$  is quite high within the range of low  $k_{te}$  values. If thermoelectric materials having low  $k_{te}$  values can, indeed, be fabricated, it appears possible for the industry to design a hybrid system that is capable of yielding high values of thermal efficiencies.

With the advances on thermoelectric research, such as the enhancement of power factor (electrical conductivity times square of Seebeck coefficient) through optimizing transport distribution functions [16] or nanostructure designs [17,18], and the reduction of thermal conductivity by nanostructure scattering [17,19] or strain engineering [20], the efficiency of thermoelectric energy conversion can be greatly increased. The PV–TE hybrid system can thus be potentially advantageous due to such significant improvements of thermoelectric materials.

It is worth mentioning as a tender reminder that, in our analysis, the power output of the solar-cell module is approximated by a linear function of the solar-cell temperature, with constants,  $d_1$  and  $d_2$ , comprehensively computed according to Eqs. (7)–(12). The benefit of this approximation is that the power output is reliably coupled with the nodal temperatures of the PV–TE hybrid system.

## 5. Conclusions

In this study, we have emphasized on coupling the temperature field with the power outputs of the hybrid system that consists of a photovoltaic solar-cell module and a thermoelectric module. The efficiency of the thermoelectric module is directly computed based on thermoelectric theories, and the value of the external electrical resistance is taken to be the one corresponding to the maximum value of efficiency. Primary findings can be summarized below:

1. According to our findings within the parametric values chosen, the thermal efficiency of the hybrid system is generally lower than that of the solar-cell module alone. If we are able to fabricate thermoelectric materials with low-valued thermal conductivities and high-valued Seebeck coefficients, it may be possible for us to achieve  $\eta_{\text{hybrid}} > \eta_{\text{pv only}}$ .
2. The power output of the solar cell module can be approximated *a priori* by a linear function of the temperature  $T(4)$ , which is then coupled with other nodal temperatures. Such an approximation greatly facilitates the thermal analysis.
3. A numerical procedure has been developed to couple the power outputs of both the photovoltaic solar-cell module and the thermoelectric module with the temperature distribution of the hybrid system. It serves as a tool for the future research in designing PV–TE hybrid assemblies.

## Conflict of Interest

All authors hold absolutely no conflict of interest toward this submitted paper, if published.

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## Appendix A

$e_1$	$2.5 \times 10^4$	
$e_2$	$5.0 \times 10^4$	
$e_3$	$2.5 \times 10^4$	
$r_0$	$1.0 \times 10^{-5} \Omega$	
$\beta$	$1.0 \times 10^{-7} \Omega/K$	
$r(1)$	$e_1(r_0 + \beta T_1)$	internal electrical resistance
$V(1)$	$nS(T_{1.5} - T_1)$	voltage
$T_{1.5}$	$(T_1 + T_2)/2$	midpoint nodal temperature
$r(2)$	$e_2(r_0 + \beta T_1)$	internal electrical resistance
$V(2)$	$nS(T_{2.5} - T_{1.5})$	voltage
$T_{2.5}$	$(T_2 + T_3)/2$	midpoint nodal temperature
$r(3)$	$e_3(r_0 + \beta T_1)$	internal electrical resistance
$V(3)$	$nS(T_3 - T_{2.5})$	voltage

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